

CONTENTS

PART A: INTRODUCTION AND BASIC TECHNIQUES	1
CHAPTER I. SOME IMPORTANT CLASSES OF GLOBAL OPTIMIZATION PROBLEMS	3
1. Global Optimization	3
2. Concave Minimization	9
2.1. Definition and Basic Properties	9
2.2. Brief Survey of Direct Applications	12
2.3. Integer Programming and Concave Minimization	14
2.4. Bilinear Programming and Concave Minimization	19
2.5. Complementarity Problems and Concave Minimization	23
2.6. Max–Min Problems and Concave Minimization	25
3. D.C. Programming and Reverse Convex Constraints	26
3.1. D.C. Programming: Basic Properties	26
3.2. D.C. Programming: Applications	32
3.3. Reverse Convex Constraints	36
3.4. Canonical D.C. Programming Problems	39
4. Lipschitzian Optimization and Systems of Equations and Inequalities	42
4.1. Lipschitzian Optimization	42
4.2. Systems of Equations and Inequalities	46
CHAPTER II. OUTER APPROXIMATION	51
1. Basic Outer Approximation Method	51
2. Outer Approximation by Convex Polyhedral Sets	56
3. Constraint Dropping Strategies	65
4. On Solving the Subproblems (Q_k)	68
4.1. Finding an Initial Polytope D_1 and its Vertex Set V_1	69
4.2. Computing New Vertices and New Extreme Directions	71
4.3. Identifying Redundant Constraints	82

CHAPTER III. CONCAVITY CUTS	85
1. Concept of a Valid Cut	85
2. Valid Cuts in the Degenerate Case	91
3. Convergence of Cutting Procedures	95
4. Concavity Cuts for Handling Reverse Convex Constraints	100
5. A Class of Generalized Concavity Cuts	104
6. Cuts Using Negative Edge Extensions	108
CHAPTER IV. BRANCH AND BOUND	111
1. A Prototype Branch and Bound Method	111
2. Finiteness and Convergence Conditions	121
3. Typical Partition Sets and their Refinement	132
3.1. Simplices	132
3.2. Rectangles and Polyhedral Cones	137
4. Lower Bounds	139
4.1. Lipschitzian Optimization	140
4.2. Vertex Minima	141
4.3. Convex Subfunctionals	142
4.4. Duality	153
4.5. Consistency	158
5. Deletion by Infeasibility	163
6. Restart Branch and Bound Algorithm	169
PART B: CONCAVE MINIMIZATION	173
CHAPTER V. CUTTING METHODS	175
1. A Pure Cutting Algorithm	175
1.1. Valid Cuts and a Sufficient Condition for Global Optimality	176
1.2. Outline of the Method	181
2. Facial Cut Algorithm	184
2.1. The Basic Idea	184
2.2. Finding an Extreme Face of D Relative to M	186
2.3. Facial Valid Cuts	190
2.4. A Finite Cutting Algorithm	192

3. Cut and Split Algorithm	195
3.1. Partition of a Cone	196
3.2. Outline of the Method	197
3.3. Remarks	200
4. Generating Deep Cuts: The Case of Concave Quadratic Functionals	205
4.1. A Hierarchy of Valid Cuts	205
4.2. Konno's Cutting Method for Concave Quadratic Programming	211
4.3. Bilinear Programming Cuts	216
CHAPTER VI. SUCCESSIVE APPROXIMATION METHODS	219
1. Outer Approximation Algorithms	219
1.1. Linearly Constrained Problem	220
1.2. Problems with Convex Constraints	228
1.3. Reducing the Sizes of the Relaxed Problems	233
2. Inner Approximation	237
2.1. The (DG) Problem	238
2.2. The Concept of Polyhedral Annexation	239
2.3. Computing the Facets of a Polytope	241
2.4. A Polyhedral Annexation Algorithm	244
2.5. Relations to Other Methods	253
2.6. Extensions	256
3. Convex Underestimation	259
3.1. Relaxation and Successive Underestimation	260
3.2. The Falk and Hoffman Algorithm	262
3.3. Rosen's Algorithm	265
4. Concave Polyhedral Underestimation	271
4.1. Outline of the Method	271
4.2. Computation of the Concave Underestimators	273
4.3. Computation of the Nonvertical Facets	274
4.4. Polyhedral Underestimation Algorithm	277
4.5. Alternative Interpretation	279
4.6. Separable Problems	281
CHAPTER VII. SUCCESSIVE PARTITION METHODS	286
1. Conical Algorithms	286
1.1. The Normal Conical Subdivision Process	287
1.2. The Main Subroutine	289
1.3. Construction of Normal Subdivision Processes	291
1.4. The Basic NCS Process	296
1.5. The Normal Conical Algorithm	299
1.6. Remarks Concerning Implementation	303
1.7. Example	306
1.8. Alternative Variants	309

1.9. Concave Minimization with Convex Constraints	314
1.10. Unbounded Feasible Domain	319
1.11. A Class of Exhaustive Subdivision Processes	320
1.12. Exhaustive Nondegenerate Subdivision Processes	326
2. Simplicial Algorithms	333
2.1. Normal Simplicial Subdivision Processes	334
2.2. Normal Simplicial Algorithm	335
2.3. Construction of an NSS Process	337
2.4. The Basic NSS Process	339
2.5. Normal Simplicial Algorithm for Problems with Convex Constraints	341
3. An Exact Simplicial Algorithm	344
3.1. Simplicial Subdivision of a Polytope	344
3.2. A Finite Branch and Bound Procedure	346
3.3. A Modified ES Algorithm	348
3.4. Unbounded Feasible Set	352
4. Rectangular Algorithms	355
4.1. Normal Rectangular Algorithm	357
4.2. Construction of an NRS Process	359
4.3. Specialization to Concave Quadratic Programming	362
4.4. Example	367
CHAPTER VIII. DECOMPOSITION OF LARGE SCALE PROBLEMS	371
1. Decomposition Framework	372
2. Branch and Bound Approach	374
2.1. Normal Simplicial Algorithm	375
2.2. Normal Rectangular Algorithm	378
2.3. Normal Conical Algorithm	380
3. Polyhedral Underestimation Method	381
3.1. Nonseparable Problems	381
3.2. Separable Problems	383
4. Decomposition by Outer Approximation	390
4.1. Basic Idea	391
4.2. Decomposition Algorithm	392
4.3. An Extension	398
4.4. Outer Approximation Versus Successive Partition	402
4.5. Outer Approximation Combined with Branch and Bound	406
5. Decomposition of Concave Minimization Problems over Networks	410
5.1. The Minimum Concave Cost Flow Problem	410
5.2. The Single Source Uncapacitated Minimum Concave Cost Flow Problem (SUCF)	414
5.3. Decomposition Method for (SUCF)	420
5.4. Extension	430

CHAPTER IX. SPECIAL PROBLEMS OF CONCAVE MINIMIZATION	434
1. Bilinear Programming	434
1.1. Basic Properties	435
1.2. Cutting Plane Method	438
1.3. Polyhedral Annexation	443
1.4. Conical Algorithm	445
1.5. Outer Approximation Method	449
2. Complementarity Problems	456
2.1. Basic Properties	457
2.2. Polyhedral Annexation Method for the Linear Complementarity Problem (LCP)	459
2.3. Conical Algorithm for the (LCP)	462
2.4. Other Global Optimization Approaches to (LCP)	470
2.5. The Concave Complementarity Problem	473
3. Parametric Concave Programming	476
3.1. Basic Properties	478
3.2. Outer Approximation Method for (LRCP)	484
3.3. Methods Based on the Edge Property	487
3.4. Conical Algorithms for (LRCP)	494
 PART C: GENERAL NONLINEAR PROBLEMS	 503
 CHAPTER X. D.C. PROGRAMMING	 505
1. Outer Approximation Methods for Solving the Canonical D.C. Programming Problem	505
1.1. Duality between the Objective and the Constraints	506
1.2. Outer Approximation Algorithms for Canonical D.C. Problems	512
1.3. Outer Approximation for Solving Noncanonical D.C. Problems	527
2. Branch and Bound Methods	539
3. Solving D.C. Problems by a Sequence of Linear Programs and Line Searches	544
4. Some Special D.C. Problems and Applications	558
4.1. The Design Centering Problem	558
4.2. The Diamond Cutting Problem	567
4.3. Biconvex Programming and Related Problems	577

CHAPTER XI. LIPSCHITZ AND CONTINUOUS OPTIMIZATION	587
1. Brief Introduction into the Global Minimization of Univariate Lipschitz Functions	588
1.1. Saw-Tooth Covers	588
1.2. Algorithms for Solving the Univariate Lipschitz-Problem	593
2. Branch and Bound Algorithms	600
2.1. Branch and Bound Interpretation of Piyavskii's Univariate Algorithm	601
2.2. Branch and Bound Methods for Minimizing a Lipschitz Function over an n -dimensional Rectangle	605
2.3. Branch and Bound Methods for Solving Lipschitz Optimization Problems with General Constraints	616
2.4. Global Optimization of Concave Functions Subject to Separable Quadratic Constraints	617
3. Outer Approximation	629
4. The Relief Indicator Method	638
4.1. Separators for f on D	638
4.2. A Global Optimality Criterion	642
4.3. The Relief Indicator Method	646
References	657
Notation	691
Index	694