
Contents

List of Algorithms	ix
Preface	xi
CHAPTER 1. Introduction	1
1.1 Brief Overview of the State of the Art	3
1.1.1 Hermitian Matrices	3
1.1.2 Non-Hermitian Matrices	5
1.1.3 Preconditioners	6
1.2 Notation	6
1.3 Review of Relevant Linear Algebra	7
1.3.1 Vector Norms and Inner Products	7
1.3.2 Orthogonality	8
1.3.3 Matrix Norms	9
1.3.4 The Spectral Radius	11
1.3.5 Canonical Forms and Decompositions	13
1.3.6 Eigenvalues and the Field of Values	16
 I Krylov Subspace Approximations	 23
CHAPTER 2. Some Iteration Methods	25
2.1 Simple Iteration	25
2.2 Orthomin(1) and Steepest Descent	29
2.3 Orthomin(2) and CG	33
2.4 Orthodir, MINRES, and GMRES	37
2.5 Derivation of MINRES and CG from the Lanczos Algorithm . .	41
 CHAPTER 3. Error Bounds for CG, MINRES, and GMRES	 49
3.1 Hermitian Problems—CG and MINRES	49
3.2 Non-Hermitian Problems—GMRES	54

CHAPTER 4. Effects of Finite Precision Arithmetic	61
4.1 Some Numerical Examples	62
4.2 The Lanczos Algorithm	63
4.3 A Hypothetical MINRES/CG Implementation	64
4.4 A Matrix Completion Problem	66
4.4.1 Paige's Theorem	67
4.4.2 A Different Matrix Completion	68
4.5 Orthogonal Polynomials	71
CHAPTER 5. BiCG and Related Methods	77
5.1 The Two-Sided Lanczos Algorithm	77
5.2 The Biconjugate Gradient Algorithm	79
5.3 The Quasi-Minimal Residual Algorithm	80
5.4 Relation Between BiCG and QMR	84
5.5 The Conjugate Gradient Squared Algorithm	88
5.6 The BiCGSTAB Algorithm	90
5.7 Which Method Should I Use?	92
CHAPTER 6. Is There a Short Recurrence for a Near-Optimal Approximation?	97
6.1 The Faber and Manteuffel Result	97
6.2 Implications	102
CHAPTER 7. Miscellaneous Issues	105
7.1 Symmetrizing the Problem	105
7.2 Error Estimation and Stopping Criteria	107
7.3 Attainable Accuracy	109
7.4 Multiple Right-Hand Sides and Block Methods	113
7.5 Computer Implementation	115
II Preconditioners	117
CHAPTER 8. Overview and Preconditioned Algorithms	119
CHAPTER 9. Two Example Problems	125
9.1 The Diffusion Equation	125
9.1.1 Poisson's Equation	129
9.2 The Transport Equation	134
CHAPTER 10. Comparison of Preconditioners	147
10.1 Jacobi, Gauss-Seidel, SOR	147
10.1.1 Analysis of SOR	149
10.2 The Perron-Frobenius Theorem	156
10.3 Comparison of Regular Splittings	160
10.4 Regular Splittings Used with the CG Algorithm	163

10.5 Optimal Diagonal and Block Diagonal Preconditioners	165
CHAPTER 11. Incomplete Decompositions	171
11.1 Incomplete Cholesky Decomposition	171
11.2 Modified Incomplete Cholesky Decomposition	175
CHAPTER 12. Multigrid and Domain Decomposition Methods	183
12.1 Multigrid Methods	183
12.1.1 Aggregation Methods	184
12.1.2 Analysis of a Two-Grid Method for the Model Problem.	187
12.1.3 Extension to More General Finite Element Equations. .	193
12.1.4 Multigrid Methods	193
12.1.5 Multigrid as a Preconditioner for Krylov Subspace Methods	197
12.2 Basic Ideas of Domain Decomposition Methods	197
12.2.1 Alternating Schwarz Method	198
12.2.2 Many Subdomains and the Use of Coarse Grids	201
12.2.3 Nonoverlapping Subdomains	203
References	205
Index	213