

# Contents

---

Preface xii  
About the Author xxii

## CHAPTER 1 The Nature of Econometrics and Economic Data 1

---

- 1-1 What Is Econometrics? 1
- 1-2 Steps in Empirical Economic Analysis 2
- 1-3 The Structure of Economic Data 5
  - 1-3a Cross-Sectional Data 5
  - 1-3b Time Series Data 7
  - 1-3c Pooled Cross Sections 8
  - 1-3d Panel or Longitudinal Data 9
  - 1-3e A Comment on Data Structures 10
- 1-4 Causality, Ceteris Paribus, and Counterfactual Reasoning 10
- Summary 14
- Key Terms 15
- Problems 15
- Computer Exercises 15

## PART 1

---

## Regression Analysis with Cross-Sectional Data 19

### CHAPTER 2 The Simple Regression Model 20

---

- 2-1 Definition of the Simple Regression Model 20
- 2-2 Deriving the Ordinary Least Squares Estimates 24
  - 2-2a A Note on Terminology 31
- 2-3 Properties of OLS on Any Sample of Data 32
  - 2-3a Fitted Values and Residuals 32
  - 2-3b Algebraic Properties of OLS Statistics 32
  - 2-3c Goodness-of-Fit 35

- 2-4 Units of Measurement and Functional Form 36
  - 2-4a The Effects of Changing Units of Measurement on OLS Statistics 36
  - 2-4b Incorporating Nonlinearities in Simple Regression 37
  - 2-4c The Meaning of "Linear" Regression 40
- 2-5 Expected Values and Variances of the OLS Estimators 40
  - 2-5a Unbiasedness of OLS 40
  - 2-5b Variances of the OLS Estimators 45
  - 2-5c Estimating the Error Variance 48
- 2-6 Regression through the Origin and Regression on a Constant 50
- 2-7 Regression on a Binary Explanatory Variable 51
  - 2-7a Counterfactual Outcomes, Causality, and Policy Analysis 53
- Summary 56
- Key Terms 57
- Problems 58
- Computer Exercises 62

### CHAPTER 3 Multiple Regression Analysis: Estimation 66

---

- 3-1 Motivation for Multiple Regression 67
  - 3-1a The Model with Two Independent Variables 67
  - 3-1b The Model with k Independent Variables 69
- 3-2 Mechanics and Interpretation of Ordinary Least Squares 70
  - 3-2a Obtaining the OLS Estimates 70
  - 3-2b Interpreting the OLS Regression Equation 71
  - 3-2c On the Meaning of "Holding Other Factors Fixed" in Multiple Regression 73
  - 3-2d Changing More Than One Independent Variable Simultaneously 74

3-2e	OLS Fitted Values and Residuals	74
3-2f	A "Partialling Out" Interpretation of Multiple Regression	75
3-2g	Comparison of Simple and Multiple Regression Estimates	75
3-2h	Goodness-of-Fit	76
3-2i	Regression through the Origin	79
3-3	The Expected Value of the OLS Estimators	79
3-3a	Including Irrelevant Variables in a Regression Model	83
3-3b	Omitted Variable Bias: The Simple Case	84
3-3c	Omitted Variable Bias: More General Cases	87
3-4	The Variance of the OLS Estimators	87
3-4a	The Components of the OLS Variances: Multicollinearity	89
3-4b	Variances in Misspecified Models	92
3-4c	Estimating $\sigma^2$ : Standard Errors of the OLS Estimators	93
3-5	Efficiency of OLS: The Gauss-Markov Theorem	95
3-6	Some Comments on the Language of Multiple Regression Analysis	96
3-7	Several Scenarios for Applying Multiple Regression	97
3-7a	Prediction	98
3-7b	Efficient Markets	98
3-7c	Measuring the Tradeoff between Two Variables	99
3-7d	Testing for <i>Ceteris Paribus</i> Group Differences	99
3-7e	Potential Outcomes, Treatment Effects, and Policy Analysis	100
	Summary	102
	Key Terms	104
	Problems	104
	Computer Exercises	109

## **CHAPTER 4 Multiple Regression Analysis: Inference 117**

---

4-1	Sampling Distributions of the OLS Estimators	117
4-2	Testing Hypotheses about a Single Population Parameter: The <i>t</i> Test	120
4-2a	Testing against One-Sided Alternatives	122
4-2b	Two-Sided Alternatives	126
4-2c	Testing Other Hypotheses about $\beta_j$	128
4-2d	Computing p-Values for <i>t</i> Tests	130

4-2e	A Reminder on the Language of Classical Hypothesis Testing	132
4-2f	Economic, or Practical, versus Statistical Significance	132
4-3	Confidence Intervals	134
4-4	Testing Hypotheses about a Single Linear Combination of the Parameters	136
4-5	Testing Multiple Linear Restrictions: The <i>F</i> Test	139
4-5a	Testing Exclusion Restrictions	139
4-5b	Relationship between <i>F</i> and <i>t</i> Statistics	144
4-5c	The R-Squared Form of the <i>F</i> Statistic	145
4-5d	Computing p-Values for <i>F</i> Tests	146
4-5e	The <i>F</i> Statistic for Overall Significance of a Regression	147
4-5f	Testing General Linear Restrictions	148
4-6	Reporting Regression Results	149
4-7	Revisiting Causal Effects and Policy Analysis	151
	Summary	152
	Key Terms	154
	Problems	154
	Computer Exercises	159

## **CHAPTER 5 Multiple Regression Analysis: OLS Asymptotics 163**

---

5-1	Consistency	164
5-1a	Deriving the Inconsistency in OLS	167
5-2	Asymptotic Normality and Large Sample Inference	168
5-2a	Other Large Sample Tests: The Lagrange Multiplier Statistic	172
5-3	Asymptotic Efficiency of OLS	175
	Summary	176
	Key Terms	176
	Problems	176
	Computer Exercises	178

## **CHAPTER 6 Multiple Regression Analysis: Further Issues 181**

---

6-1	Effects of Data Scaling on OLS Statistics	181
6-1a	Beta Coefficients	184
6-2	More on Functional Form	186
6-2a	More on Using Logarithmic Functional Forms	186

6-2b	<i>Models with Quadratics</i>	188
6-2c	<i>Models with Interaction Terms</i>	192
6-2d	<i>Computing Average Partial Effects</i>	194
6-3	More on Goodness-of-Fit and Selection of Regressors	195
6-3a	<i>Adjusted R-Squared</i>	196
6-3b	<i>Using Adjusted R-Squared to Choose between Nonnested Models</i>	197
6-3c	<i>Controlling for Too Many Factors in Regression Analysis</i>	199
6-3d	<i>Adding Regressors to Reduce the Error Variance</i>	200
6-4	Prediction and Residual Analysis	201
6-4a	<i>Confidence Intervals for Predictions</i>	201
6-4b	<i>Residual Analysis</i>	205
6-4c	<i>Predicting y When log(y) Is the Dependent Variable</i>	205
6-4d	<i>Predicting y When the Dependent Variable Is log(y)</i>	207
	Summary	209
	Key Terms	211
	Problems	211
	Computer Exercises	214

## CHAPTER 7 Multiple Regression Analysis with Qualitative Information 220

7-1	Describing Qualitative Information	221
7-2	A Single Dummy Independent Variable	222
7-2a	<i>Interpreting Coefficients on Dummy Explanatory Variables When the Dependent Variable Is log(y)</i>	226
7-3	Using Dummy Variables for Multiple Categories	228
7-3a	<i>Incorporating Ordinal Information by Using Dummy Variables</i>	230
7-4	Interactions Involving Dummy Variables	232
7-4a	<i>Interactions among Dummy Variables</i>	232
7-4b	<i>Allowing for Different Slopes</i>	233
7-4c	<i>Testing for Differences in Regression Functions across Groups</i>	237
7-5	A Binary Dependent Variable: The Linear Probability Model	239
7-6	More on Policy Analysis and Program Evaluation	244
7-6a	<i>Program Evaluation and Unrestricted Regression Adjustment</i>	245

7-7	Interpreting Regression Results with Discrete Dependent Variables	249
-----	---	-----

Summary	250
Key Terms	251
Problems	251
Computer Exercises	256

## CHAPTER 8 Heteroskedasticity 262

8-1	Consequences of Heteroskedasticity for OLS	262
8-2	Heteroskedasticity-Robust Inference after OLS Estimation	263
8-2a	<i>Computing Heteroskedasticity-Robust LM Tests</i>	267
8-3	Testing for Heteroskedasticity	269
8-3a	<i>The White Test for Heteroskedasticity</i>	271
8-4	Weighted Least Squares Estimation	273
8-4a	<i>The Heteroskedasticity Is Known up to a Multiplicative Constant</i>	273
8-4b	<i>The Heteroskedasticity Function Must Be Estimated: Feasible GLS</i>	278
8-4c	<i>What If the Assumed Heteroskedasticity Function Is Wrong?</i>	281
8-4d	<i>Prediction and Prediction Intervals with Heteroskedasticity</i>	283
8-5	The Linear Probability Model Revisited	284
	Summary	286
	Key Terms	287
	Problems	287
	Computer Exercises	290

## CHAPTER 9 More on Specification and Data Issues 294

9-1	Functional Form Misspecification	295
9-1a	<i>RESET as a General Test for Functional Form Misspecification</i>	297
9-1b	<i>Tests against Nonnested Alternatives</i>	298
9-2	Using Proxy Variables for Unobserved Explanatory Variables	299
9-2a	<i>Using Lagged Dependent Variables as Proxy Variables</i>	303
9-2b	<i>A Different Slant on Multiple Regression</i>	304
9-2c	<i>Potential Outcomes and Proxy Variables</i>	305
9-3	Models with Random Slopes	306
9-4	Properties of OLS under Measurement Error	308
9-4a	<i>Measurement Error in the Dependent Variable</i>	308

- 9-4b *Measurement Error in an Explanatory Variable* 310
- 9-5 Missing Data, Nonrandom Samples, and Outlying Observations 313
  - 9-5a *Missing Data* 313
  - 9-5b *Nonrandom Samples* 315
  - 9-5c *Outliers and Influential Observations* 317
- 9-6 Least Absolute Deviations Estimation 321
- Summary 323
- Key Terms 324
- Problems 324
- Computer Exercises 328

## PART 2

## Regression Analysis with Time Series Data 333

### CHAPTER 10 Basic Regression Analysis with Time Series Data 334

- 10-1 The Nature of Time Series Data 334
- 10-2 Examples of Time Series Regression Models 335
  - 10-2a *Static Models* 336
  - 10-2b *Finite Distributed Lag Models* 336
  - 10-2c *A Convention about the Time Index* 338
- 10-3 Finite Sample Properties of OLS under Classical Assumptions 339
  - 10-3a *Unbiasedness of OLS* 339
  - 10-3b *The Variances of the OLS Estimators and the Gauss-Markov Theorem* 342
  - 10-3c *Inference under the Classical Linear Model Assumptions* 344
- 10-4 Functional Form, Dummy Variables, and Index Numbers 345
- 10-5 Trends and Seasonality 351
  - 10-5a *Characterizing Trending Time Series* 351
  - 10-5b *Using Trending Variables in Regression Analysis* 354
  - 10-5c *A Detrending Interpretation of Regressions with a Time Trend* 356
  - 10-5d *Computing R-Squared When the Dependent Variable Is Trending* 357
  - 10-5e *Seasonality* 358
- Summary 360
- Key Terms 361

- Problems 361
- Computer Exercises 363

### CHAPTER 11 Further Issues in Using OLS with Time Series Data 366

- 11-1 Stationary and Weakly Dependent Time Series 367
  - 11-1a *Stationary and Nonstationary Time Series* 367
  - 11-1b *Weakly Dependent Time Series* 368
- 11-2 Asymptotic Properties of OLS 370
- 11-3 Using Highly Persistent Time Series in Regression Analysis 376
  - 11-3a *Highly Persistent Time Series* 376
  - 11-3b *Transformations on Highly Persistent Time Series* 380
  - 11-3c *Deciding Whether a Time Series Is I(1)* 381
- 11-4 Dynamically Complete Models and the Absence of Serial Correlation 382
- 11-5 The Homoskedasticity Assumption for Time Series Models 385
- Summary 386
- Key Terms 387
- Problems 387
- Computer Exercises 390

### CHAPTER 12 Serial Correlation and Heteroskedasticity in Time Series Regressions 394

- 12-1 Properties of OLS with Serially Correlated Errors 395
  - 12-1a *Unbiasedness and Consistency* 395
  - 12-1b *Efficiency and Inference* 395
  - 12-1c *Goodness-of-Fit* 396
  - 12-1d *Serial Correlation in the Presence of Lagged Dependent Variables* 396
- 12-2 Serial Correlation—Robust Inference after OLS 398
- 12-3 Testing for Serial Correlation 401
  - 12-3a *A t Test for AR(1) Serial Correlation with Strictly Exogenous Regressors* 402
  - 12-3b *The Durbin-Watson Test under Classical Assumptions* 403
  - 12-3c *Testing for AR(1) Serial Correlation without Strictly Exogenous Regressors* 404
  - 12-3d *Testing for Higher-Order Serial Correlation* 406

**12-4 Correcting for Serial Correlation with Strictly Exogenous Regressors 407**12-4a *Obtaining the Best Linear Unbiased Estimator in the AR(1) Model* 40812-4b *Feasible GLS Estimation with AR(1) Errors* 40912-4c *Comparing OLS and FGLS* 41112-4d *Correcting for Higher-Order Serial Correlation* 41312-4e *What if the Serial Correlation Model Is Wrong?* 413**12-5 Differencing and Serial Correlation 414****12-6 Heteroskedasticity in Time Series Regressions 415**12-6a *Heteroskedasticity-Robust Statistics* 41612-6b *Testing for Heteroskedasticity* 41612-6c *Autoregressive Conditional Heteroskedasticity* 41712-6d *Heteroskedasticity and Serial Correlation in Regression Models* 418

Summary 419

Key Terms 420

Problems 420

Computer Exercises 421

**PART 3****Advanced Topics 425****CHAPTER 13 Pooling Cross Sections across Time: Simple Panel Data Methods 426****13-1 Pooling Independent Cross Sections across Time 427**13-1a *The Chow Test for Structural Change across Time* 431**13-2 Policy Analysis with Pooled Cross Sections 431**13-2a *Adding an Additional Control Group* 43613-2b *A General Framework for Policy Analysis with Pooled Cross Sections* 437**13-3 Two-Period Panel Data Analysis 439**13-3a *Organizing Panel Data* 444**13-4 Policy Analysis with Two-Period Panel Data 444****13-5 Differencing with More Than Two Time Periods 447**13-5a *Potential Pitfalls in First Differencing Panel Data* 451

Summary 451

Key Terms 452

Problems 452

Computer Exercises 453

**CHAPTER 14 Advanced Panel Data Methods 462****14-1 Fixed Effects Estimation 463**14-1a *The Dummy Variable Regression* 46614-1b *Fixed Effects or First Differencing?* 46714-1c *Fixed Effects with Unbalanced Panels* 468**14-2 Random Effects Models 469**14-2a *Random Effects or Pooled OLS?* 47314-2b *Random Effects or Fixed Effects?* 473**14-3 The Correlated Random Effects Approach 474**14-3a *Unbalanced Panels* 476**14-4 General Policy Analysis with Panel Data 477**14-4a *Advanced Considerations with Policy Analysis* 478**14-5 Applying Panel Data Methods to Other Data Structures 480**

Summary 483

Key Terms 484

Problems 484

Computer Exercises 486

**CHAPTER 15 Instrumental Variables Estimation and Two-Stage Least Squares 495****15-1 Motivation: Omitted Variables in a Simple Regression Model 496**15-1a *Statistical Inference with the IV Estimator* 50015-1b *Properties of IV with a Poor Instrumental Variable* 50315-1c *Computing R-Squared after IV Estimation* 505**15-2 IV Estimation of the Multiple Regression Model 505****15-3 Two-Stage Least Squares 509**15-3a *A Single Endogenous Explanatory Variable* 50915-3b *Multicollinearity and 2SLS* 51115-3c *Detecting Weak Instruments* 51215-3d *Multiple Endogenous Explanatory Variables* 51315-3e *Testing Multiple Hypotheses after 2SLS Estimation* 513

15-4	IV Solutions to Errors-in-Variables Problems	514
15-5	Testing for Endogeneity and Testing Overidentifying Restrictions	515
15-5a	Testing for Endogeneity	515
15-5b	Testing Overidentification Restrictions	516
15-6	2SLS with Heteroskedasticity	518
15-7	Applying 2SLS to Time Series Equations	519
15-8	Applying 2SLS to Pooled Cross Sections and Panel Data	521
	Summary	522
	Key Terms	523
	Problems	523
	Computer Exercises	526

## CHAPTER 16 Simultaneous Equations Models 534

---

16-1	The Nature of Simultaneous Equations Models	535
16-2	Simultaneity Bias in OLS	538
16-3	Identifying and Estimating a Structural Equation	539
16-3a	Identification in a Two-Equation System	540
16-3b	Estimation by 2SLS	543
16-4	Systems with More Than Two Equations	545
16-4a	Identification in Systems with Three or More Equations	545
16-4b	Estimation	546
16-5	Simultaneous Equations Models with Time Series	546
16-6	Simultaneous Equations Models with Panel Data	549
	Summary	551
	Key Terms	552
	Problems	552
	Computer Exercises	555

## CHAPTER 17 Limited Dependent Variable Models and Sample Selection Corrections 559

---

17-1	Logit and Probit Models for Binary Response	560
17-1a	Specifying Logit and Probit Models	560
17-1b	Maximum Likelihood Estimation of Logit and Probit Models	563
17-1c	Testing Multiple Hypotheses	564
17-1d	Interpreting the Logit and Probit Estimates	565

17-2	The Tobit Model for Corner Solution Responses	571
17-2a	Interpreting the Tobit Estimates	572
17-2b	Specification Issues in Tobit Models	578
17-3	The Poisson Regression Model	578
17-4	Censored and Truncated Regression Models	582
17-4a	Censored Regression Models	583
17-4b	Truncated Regression Models	586
17-5	Sample Selection Corrections	588
17-5a	When Is OLS on the Selected Sample Consistent?	588
17-5b	Incidental Truncation	589
	Summary	593
	Key Terms	593
	Problems	594
	Computer Exercises	596

## CHAPTER 18 Advanced Time Series Topics 604

---

18-1	Infinite Distributed Lag Models	605
18-1a	The Geometric (or Koyck) Distributed Lag Model	607
18-1b	Rational Distributed Lag Models	608
18-2	Testing for Unit Roots	610
18-3	Spurious Regression	614
18-4	Cointegration and Error Correction Models	616
18-4a	Cointegration	616
18-4b	Error Correction Models	620
18-5	Forecasting	622
18-5a	Types of Regression Models Used for Forecasting	623
18-5b	One-Step-Ahead Forecasting	624
18-5c	Comparing One-Step-Ahead Forecasts	627
18-5d	Multiple-Step-Ahead Forecasts	628
18-5e	Forecasting Trending, Seasonal, and Integrated Processes	631
	Summary	635
	Key Terms	636
	Problems	636
	Computer Exercises	638

## CHAPTER 19 Carrying Out an Empirical Project 642

---

19-1	Posing a Question	642
19-2	Literature Review	644

<b>19-3 Data Collection</b>	<b>645</b>
19-3a <i>Deciding on the Appropriate Data Set</i>	645
19-3b <i>Entering and Storing Your Data</i>	646
19-3c <i>Inspecting, Cleaning, and Summarizing Your Data</i>	647
<b>19-4 Econometric Analysis</b>	<b>648</b>
<b>19-5 Writing an Empirical Paper</b>	<b>651</b>
19-5a <i>Introduction</i>	651
19-5b <i>Conceptual (or Theoretical) Framework</i>	652
19-5c <i>Econometric Models and Estimation Methods</i>	652
19-5d <i>The Data</i>	654
19-5e <i>Results</i>	655
19-5f <i>Conclusions</i>	656
19-5g <i>Style Hints</i>	656
Summary	658
Key Terms	658
Sample Empirical Projects	658
List of Journals	664
Data Sources	665

## **MATH REFRESHER A Basic Mathematical Tools 666**

---

<b>A-1 The Summation Operator and Descriptive Statistics</b>	<b>666</b>
<b>A-2 Properties of Linear Functions</b>	<b>668</b>
<b>A-3 Proportions and Percentages</b>	<b>671</b>
<b>A-4 Some Special Functions and Their Properties</b>	<b>672</b>
A-4a <i>Quadratic Functions</i>	672
A-4b <i>The Natural Logarithm</i>	674
A-4c <i>The Exponential Function</i>	677
<b>A-5 Differential Calculus</b>	<b>678</b>
Summary	680
Key Terms	681
Problems	681

## **MATH REFRESHER B Fundamentals of Probability 684**

---

<b>B-1 Random Variables and Their Probability Distributions</b>	<b>684</b>
B-1a <i>Discrete Random Variables</i>	685
B-1b <i>Continuous Random Variables</i>	687

<b>B-2 Joint Distributions, Conditional Distributions, and Independence</b>	<b>688</b>
B-2a <i>Joint Distributions and Independence</i>	688
B-2b <i>Conditional Distributions</i>	690
<b>B-3 Features of Probability Distributions</b>	<b>691</b>
B-3a <i>A Measure of Central Tendency: The Expected Value</i>	691
B-3b <i>Properties of Expected Values</i>	692
B-3c <i>Another Measure of Central Tendency: The Median</i>	694
B-3d <i>Measures of Variability: Variance and Standard Deviation</i>	695
B-3e <i>Variance</i>	695
B-3f <i>Standard Deviation</i>	696
B-3g <i>Standardizing a Random Variable</i>	696
B-3h <i>Skewness and Kurtosis</i>	697
<b>B-4 Features of Joint and Conditional Distributions</b>	<b>697</b>
B-4a <i>Measures of Association: Covariance and Correlation</i>	697
B-4b <i>Covariance</i>	697
B-4c <i>Correlation Coefficient</i>	698
B-4d <i>Variance of Sums of Random Variables</i>	699
B-4e <i>Conditional Expectation</i>	700
B-4f <i>Properties of Conditional Expectation</i>	702
B-4g <i>Conditional Variance</i>	704
<b>B-5 The Normal and Related Distributions</b>	<b>704</b>
B-5a <i>The Normal Distribution</i>	704
B-5b <i>The Standard Normal Distribution</i>	705
B-5c <i>Additional Properties of the Normal Distribution</i>	707
B-5d <i>The Chi-Square Distribution</i>	708
B-5e <i>The t Distribution</i>	708
B-5f <i>The F Distribution</i>	709
Summary	711
Key Terms	711
Problems	711

## **MATH REFRESHER C Fundamentals of Mathematical Statistics 714**

---

<b>C-1 Populations, Parameters, and Random Sampling</b>	<b>714</b>
C-1a <i>Sampling</i>	714
<b>C-2 Finite Sample Properties of Estimators</b>	<b>715</b>
C-2a <i>Estimators and Estimates</i>	715
C-2b <i>Unbiasedness</i>	716

C-2c	<i>The Sampling Variance of Estimators</i>	718
C-2d	<i>Efficiency</i>	719
<b>C-3</b>	<b>Asymptotic or Large Sample Properties of Estimators</b>	<b>721</b>
C-3a	<i>Consistency</i>	721
C-3b	<i>Asymptotic Normality</i>	723
<b>C-4</b>	<b>General Approaches to Parameter Estimation</b>	<b>724</b>
C-4a	<i>Method of Moments</i>	725
C-4b	<i>Maximum Likelihood</i>	725
C-4c	<i>Least Squares</i>	726
<b>C-5</b>	<b>Interval Estimation and Confidence Intervals</b>	<b>727</b>
C-5a	<i>The Nature of Interval Estimation</i>	727
C-5b	<i>Confidence Intervals for the Mean from a Normally Distributed Population</i>	729
C-5c	<i>A Simple Rule of Thumb for a 95% Confidence Interval</i>	731
C-5d	<i>Asymptotic Confidence Intervals for Nonnormal Populations</i>	732
<b>C-6</b>	<b>Hypothesis Testing</b>	<b>733</b>
C-6a	<i>Fundamentals of Hypothesis Testing</i>	733
C-6b	<i>Testing Hypotheses about the Mean in a Normal Population</i>	735
C-6c	<i>Asymptotic Tests for Nonnormal Populations</i>	738
C-6d	<i>Computing and Using p-Values</i>	738
C-6e	<i>The Relationship between Confidence Intervals and Hypothesis Testing</i>	741
C-6f	<i>Practical versus Statistical Significance</i>	742
<b>C-7</b>	<b>Remarks on Notation</b>	<b>743</b>
	Summary	743
	Key Terms	744
	Problems	744

## **ADVANCED TREATMENT D Summary of Matrix Algebra 749**

---

<b>D-1</b>	<b>Basic Definitions</b>	<b>749</b>
<b>D-2</b>	<b>Matrix Operations</b>	<b>750</b>
D-2a	<i>Matrix Addition</i>	750
D-2b	<i>Scalar Multiplication</i>	750
D-2c	<i>Matrix Multiplication</i>	751
D-2d	<i>Transpose</i>	752

D-2e	<i>Partitioned Matrix Multiplication</i>	752
D-2f	<i>Trace</i>	753
D-2g	<i>Inverse</i>	753
<b>D-3</b>	<b>Linear Independence and Rank of a Matrix</b>	<b>754</b>
<b>D-4</b>	<b>Quadratic Forms and Positive Definite Matrices</b>	<b>754</b>
<b>D-5</b>	<b>Idempotent Matrices</b>	<b>755</b>
<b>D-6</b>	<b>Differentiation of Linear and Quadratic Forms</b>	<b>755</b>
<b>D-7</b>	<b>Moments and Distributions of Random Vectors</b>	<b>756</b>
D-7a	<i>Expected Value</i>	756
D-7b	<i>Variance-Covariance Matrix</i>	756
D-7c	<i>Multivariate Normal Distribution</i>	756
D-7d	<i>Chi-Square Distribution</i>	757
D-7e	<i>t Distribution</i>	757
D-7f	<i>F Distribution</i>	757
	Summary	757
	Key Terms	757
	Problems	758

## **ADVANCED TREATMENT E The Linear Regression Model in Matrix Form 760**

---

<b>E-1</b>	<b>The Model and Ordinary Least Squares Estimation</b>	<b>760</b>
E-1a	<i>The Frisch-Waugh Theorem</i>	762
<b>E-2</b>	<b>Finite Sample Properties of OLS</b>	<b>763</b>
<b>E-3</b>	<b>Statistical Inference</b>	<b>767</b>
<b>E-4</b>	<b>Some Asymptotic Analysis</b>	<b>769</b>
E-4a	<i>Wald Statistics for Testing Multiple Hypotheses</i>	771
	Summary	771
	Key Terms	771
	Problems	772
	Answers to Going Further Questions	775
	Statistical Tables	784
	References	791
	Glossary	797
	Index	812