
Contents

Preface	xi
Acknowledgments	xiii
1 Brief Overview of Partial Differential Equations	1
1.1 The parabolic equations	1
1.2 The wave equations	2
1.3 The elliptic equations	3
1.4 Differential equations in broader areas	3
1.4.1 Electromagnetics	3
1.4.2 Fluid mechanics	4
1.4.3 Ground water contamination	5
1.4.4 Petroleum reservoir simulation	6
1.4.5 Finance modeling	7
1.4.6 Image processing	7
1.5 A quick review of numerical methods for PDEs	8
References	10
2 Finite Difference Methods for Parabolic Equations	13
2.1 Introduction	13
2.2 Theoretical issues: stability, consistence, and convergence . .	15
2.3 1-D parabolic equations	16
2.3.1 The θ -method	16
2.3.2 Some extensions	19
2.4 2-D and 3-D parabolic equations	23
2.4.1 Standard explicit and implicit methods	23
2.4.2 The ADI methods for 2-D problems	25
2.4.3 The ADI methods for 3-D problems	28
2.5 Numerical examples with MATLAB codes	30
2.6 Bibliographical remarks	33
2.7 Exercises	33
References	36

3	Finite Difference Methods for Hyperbolic Equations	39
3.1	Introduction	39
3.2	Some basic difference schemes	40
3.3	Dissipation and dispersion errors	42
3.4	Extensions to conservation laws	44
3.5	The second-order hyperbolic PDEs	45
3.6	Numerical examples with MATLAB codes	49
3.7	Bibliographical remarks	52
3.8	Exercises	52
	References	54
4	Finite Difference Methods for Elliptic Equations	57
4.1	Introduction	57
4.2	Numerical solution of linear systems	59
4.2.1	Direct methods	59
4.2.2	Simple iterative methods	61
4.2.3	Modern iterative methods	64
4.3	Error analysis with a maximum principle	66
4.4	Some extensions	68
4.4.1	Mixed boundary conditions	68
4.4.2	Self-adjoint problems	69
4.4.3	A fourth-order scheme	70
4.5	Numerical examples with MATLAB codes	73
4.6	Bibliographical remarks	75
4.7	Exercises	76
	References	78
5	High-Order Compact Difference Methods	79
5.1	One-dimensional problems	79
5.1.1	Spatial discretization	79
5.1.2	Approximations of high-order derivatives	83
5.1.3	Temporal discretization	92
5.1.4	Low-pass spatial filter	92
5.1.5	Numerical examples with MATLAB codes	93
5.2	High-dimensional problems	110
5.2.1	Temporal discretization for 2-D problems	110
5.2.2	Stability analysis	112
5.2.3	Extensions to 3-D compact ADI schemes	113
5.2.4	Numerical examples with MATLAB codes	114
5.3	Other high-order compact schemes	122
5.3.1	One-dimensional problems	122
5.3.2	Two-dimensional problems	124
5.4	Bibliographical remarks	127

5.5 Exercises	127
-------------------------	-----

References	130
-------------------	------------

6 Finite Element Methods: Basic Theory	133
---	------------

6.1 Introduction to one-dimensional problems	133
6.1.1 The second-order equation	133
6.1.2 The fourth-order equation	136
6.2 Introduction to two-dimensional problems	140
6.2.1 The Poisson's equation	140
6.2.2 The biharmonic problem	142
6.3 Abstract finite element theory	143
6.3.1 Existence and uniqueness	143
6.3.2 Stability and convergence	145
6.4 Examples of conforming finite element spaces	146
6.4.1 Triangular finite elements	147
6.4.2 Rectangular finite elements	149
6.5 Examples of nonconforming finite elements	150
6.5.1 Nonconforming triangular elements	150
6.5.2 Nonconforming rectangular elements	151
6.6 Finite element interpolation theory	153
6.6.1 Sobolev spaces	154
6.6.2 Interpolation theory	155
6.7 Finite element analysis of elliptic problems	159
6.7.1 Analysis of conforming finite elements	159
6.7.2 Analysis of nonconforming finite elements	161
6.8 Finite element analysis of time-dependent problems	163
6.8.1 Introduction	163
6.8.2 FEM for parabolic equations	164
6.9 Bibliographical remarks	167
6.10 Exercises	167

References	169
-------------------	------------

7 Finite Element Methods: Programming	173
--	------------

7.1 FEM mesh generation	173
7.2 Forming FEM equations	178
7.3 Calculation of element matrices	179
7.4 Assembly and implementation of boundary conditions	184
7.5 The MATLAB code for P_1 element	185
7.6 The MATLAB code for the Q_1 element	188
7.7 Bibliographical remarks	193
7.8 Exercises	194

References	197
-------------------	------------

8	Mixed Finite Element Methods	199
8.1	An abstract formulation	199
8.2	Mixed methods for elliptic problems	203
8.2.1	The mixed variational formulation	203
8.2.2	The mixed finite element spaces	205
8.2.3	The error estimates	208
8.3	Mixed methods for the Stokes problem	211
8.3.1	The mixed variational formulation	211
8.3.2	Mixed finite element spaces	212
8.4	An example MATLAB code for the Stokes problem	217
8.5	Mixed methods for viscous incompressible flows	231
8.5.1	The steady Navier-Stokes problem	231
8.5.2	The unsteady Navier-Stokes problem	233
8.6	Bibliographical remarks	234
8.7	Exercises	235
	References	237
9	Finite Element Methods for Electromagnetics	241
9.1	Introduction to Maxwell's equations	241
9.2	The time-domain finite element method	243
9.2.1	The mixed method	243
9.2.2	The standard Galerkin method	248
9.2.3	The discontinuous Galerkin method	251
9.3	The frequency-domain finite element method	256
9.3.1	The standard Galerkin method	256
9.3.2	The discontinuous Galerkin method	257
9.3.3	The mixed DG method	261
9.4	The Maxwell's equations in dispersive media	263
9.4.1	Isotropic cold plasma	264
9.4.2	Debye medium	268
9.4.3	Lorentz medium	270
9.4.4	Double-negative metamaterials	273
9.5	Bibliographical remarks	281
9.6	Exercises	281
	References	283
10	Meshless Methods with Radial Basis Functions	287
10.1	Introduction	287
10.2	The radial basis functions	288
10.3	The MFS-DRM	291
10.3.1	The fundamental solution of PDEs	291
10.3.2	The MFS for Laplace's equation	294
10.3.3	The MFS-DRM for elliptic equations	297

10.3.4	Computing particular solutions using RBFs	300
10.3.5	The RBF-MFS	302
10.3.6	The MFS-DRM for the parabolic equations	302
10.4	Kansa's method	304
10.4.1	Kansa's method for elliptic problems	304
10.4.2	Kansa's method for parabolic equations	305
10.4.3	The Hermite-Birkhoff collocation method	306
10.5	Numerical examples with MATLAB codes	308
10.5.1	Elliptic problems	308
10.5.2	Biharmonic problems	315
10.6	Coupling RBF meshless methods with DDM	322
10.6.1	Overlapping DDM	323
10.6.2	Non-overlapping DDM	324
10.6.3	One numerical example	325
10.7	Bibliographical remarks	327
10.8	Exercises	328
References		329
11 Other Meshless Methods		335
11.1	Construction of meshless shape functions	335
11.1.1	The smooth particle hydrodynamics method	335
11.1.2	The moving least-square approximation	337
11.1.3	The partition of unity method	338
11.2	The element-free Galerkin method	340
11.3	The meshless local Petrov-Galerkin method	342
11.4	Bibliographical remarks	345
11.5	Exercises	345
References		346
Appendix A Answers to Selected Problems		349
Index		361