

# Contents

<b>Foreword</b>	<b>v</b>
<b>I Formation of Singularities in the Mean Curvature Flow</b>	<b>1</b>
<i>Carlo Sinestrari</i>	
1 Introduction . . . . .	3
2 Geometry of hypersurfaces . . . . .	5
3 Examples . . . . .	9
4 Local existence and formation of singularities . . . . .	10
5 Invariance properties . . . . .	16
6 Singular behaviour of convex surfaces . . . . .	20
7 Convexity estimates . . . . .	23
8 Rescaling near a singularity . . . . .	25
9 Huisken's monotonicity formula . . . . .	28
10 Cylindrical and gradient estimates . . . . .	32
11 Mean curvature flow with surgeries . . . . .	36
Bibliography . . . . .	39
<b>II Geometric Flows, Isoperimetric Inequalities and Hyperbolic Geometry</b>	<b>45</b>
<i>Manuel Ritoré</i>	
<b>Preface</b>	<b>49</b>
<b>1 The classical isoperimetric inequality in Euclidean space</b>	<b>53</b>
1.1 Introduction . . . . .	53
1.2 Preliminaries . . . . .	53
1.2.1 Area and volume . . . . .	53
1.2.2 Variational formulas . . . . .	55
1.2.3 The isoperimetric profile . . . . .	56
1.2.4 Isoperimetric regions . . . . .	57

1.3	The isoperimetric inequality in Euclidean space . . . . .	58
1.3.1	A calibration argument . . . . .	59
1.3.2	Schwarz symmetrization . . . . .	61
1.3.3	Another proof of the isoperimetric inequality . . . . .	66
<b>2</b>	<b>Surfaces</b>	<b>69</b>
2.1	Introduction . . . . .	69
2.1.1	The Gauss–Bonnet Theorem . . . . .	70
2.2	Curve shortening flow . . . . .	72
2.2.1	Basic results . . . . .	72
2.2.2	The avoidance principle . . . . .	74
2.3	Applications of curve shortening flow to isoperimetric inequalities .	75
2.3.1	Planes . . . . .	75
2.3.2	Spheres . . . . .	82
<b>3</b>	<b>Higher dimensions</b>	<b>85</b>
3.1	Introduction . . . . .	85
3.2	$H^k$ -flows and isoperimetric inequalities . . . . .	86
3.3	Estimates on the Willmore functional and isoperimetric inequalities	89
3.3.1	Euclidean spaces . . . . .	89
3.3.2	3-dimensional Hadamard manifolds . . . . .	91
3.4	Singularities in the volume-preserving mean curvature flow . . . .	96
<b>4</b>	<b>Some applications to hyperbolic geometry</b>	<b>99</b>
4.1	Introduction . . . . .	99
4.2	Bounds on the Heegaard genus of a hyperbolic manifold . . . . .	100
4.3	The isoperimetric profile for small volumes . . . . .	102
	<b>Bibliography</b>	<b>105</b>