

# Contents

<b>1</b>	<b>Mathematical and Numerical Analysis of Some FSI Problems .....</b>	<b>1</b>
	Céline Grandmont, Mária Lukáčová-Medvid'ová, and Šárka Nečasová	
1.1	Introduction .....	1
1.1.1	Variational Formulation and Energy Estimates .....	6
1.1.2	Difficulties .....	12
1.2	Mathematical Analysis .....	13
1.2.1	A Linear Simplified Problem .....	16
1.2.2	Existence of Weak Solutions .....	19
1.2.3	Existence of Strong Solutions .....	31
1.2.4	Non-Newtonian Shear-Dependent Fluid .....	36
1.3	Numerical Analysis .....	50
1.3.1	Explicit Scheme and Added Mass Effect .....	52
1.3.2	A Semi-Implicit Scheme .....	53
1.3.3	Kinematically Coupled Schemes .....	58
1.4	Conclusions .....	69
	References .....	69
<b>2</b>	<b>Fluid–Structure Interaction in Hemodynamics: Modeling, Analysis, and Numerical Simulation .....</b>	<b>79</b>
	Sunčica Čanić, Boris Muha, and Martina Bukač	
2.1	Introduction .....	80
2.2	Mathematical Models of Arterial Walls .....	83
2.2.1	Elastodynamics of Thin Structures .....	84
2.2.2	Elastodynamics of Structures with Finite Thickness (“Thick Structures”) .....	100
2.3	A Benchmark Problem .....	104
2.3.1	The Model Equations .....	104
2.3.2	The Coupling Conditions .....	106
2.3.3	The Boundary and Initial Conditions .....	107

2.4	FSI Literature Review .....	109
2.4.1	Literature on Analysis of FSI Problems .....	110
2.4.2	Literature on Numerical Simulation of FSI Problems.....	111
2.5	Solution Framework .....	113
2.5.1	The Energy of the Coupled Problem .....	114
2.5.2	ALE Formulation .....	118
2.5.3	The Splitting Scheme: General Framework .....	120
2.5.4	A Modified Splitting Scheme Achieving Higher Accuracy .....	124
2.6	Existence of a Weak Solution .....	125
2.6.1	Problem Definition .....	126
2.6.2	The Energy of the Coupled Problem .....	129
2.6.3	The ALE Formulation and Lie Splitting .....	130
2.6.4	Weak Solutions .....	134
2.6.5	Approximate Solutions .....	138
2.6.6	Convergence of Approximate Solutions .....	144
2.6.7	The Limiting Problem and Weak Solution.....	157
2.6.8	Construction of the Appropriate Test Functions.....	157
2.7	Numerical Simulation .....	167
2.7.1	Problem Definition .....	167
2.7.2	The Energy of the Coupled Problem .....	170
2.7.3	The ALE Formulation .....	171
2.7.4	Weak Formulation of FSI Problem (2.177)–(2.191) .....	173
2.7.5	Numerical Implementation of the Splitting Scheme .....	174
2.7.6	Discretized Scheme in Weak Form.....	177
2.7.7	Numerical Examples .....	180
2.8	Conclusions .....	187
	References .....	188
<b>3</b>	<b>Hyperbolic–Parabolic Coupling and the Occurrence of Resonance in Partially Dissipative Systems .....</b>	<b>197</b>
	Giovanni Paolo Galdi, Mahdi Mohebbi, Rana Zakerzadeh, and Paolo Zunino	
3.1	Introduction .....	197
3.2	Resonance in a Linearized Elastic Solid.....	202
3.3	An Interesting Case Study: Linear Thermoelasticity .....	206
3.3.1	The One-Dimensional Case .....	208
3.3.2	The Higher Dimensional Case .....	213
3.4	An Abstract Approach .....	217
3.5	Some Applications.....	231
3.5.1	Three-Dimensional Linear Thermoelasticity (Revisited) .....	231
3.5.2	Linear Magnetoelasticity .....	233
3.5.3	A Liquid–Structure Interaction Problem Showing Generic Absence of Resonance .....	237

3.5.4	Interaction of a Viscous Liquid with a Thin Structure: The Flat Case .....	241
3.5.5	Interaction of a Viscous Liquid with a Thin Structure: The Curved Case .....	244
3.6	Numerical Experiments .....	249
	References .....	254
<b>4</b>	<b>Topics in the Mathematical Theory of Interactions of Incompressible Viscous Fluid with Rigid Bodies .....</b>	<b>257</b>
	Matthieu Hillairet	
4.1	Introduction .....	257
4.2	Basic Equations .....	258
4.2.1	Notations .....	258
4.2.2	Equations of Motion .....	260
4.2.3	Outline of the Paper .....	262
4.3	Existence and Uniqueness for the Initial Boundary-Value Problems .....	262
4.3.1	Many Bodies in a Container: Weak Solutions .....	263
4.3.2	Motion of One Body in a Bounded Domain: Classical Solutions .....	281
4.4	The Contact Issue .....	292
4.4.1	Blow-Up Criterion for Classical Solutions .....	292
4.4.2	On Weak Solutions with Contact .....	299
4.4.3	Contact vs No-Contact .....	305
	References .....	318
<b>5</b>	<b>Numerical Simulation of Fluid–Structure Interaction Problems with Applications to Flow in Vocal Folds .....</b>	<b>321</b>
	Miloslav Feistauer, Petr Sváček, and Jaromír Horáček	
5.1	Introduction .....	322
5.2	Incompressible Flow in Time-Dependent Domains .....	327
5.2.1	Incompressible Navier–Stokes Equations .....	328
5.2.2	Arbitrary Lagrangian–Eulerian Method .....	329
5.2.3	Numerical Approximation of the Incompressible Navier–Stokes Equations .....	330
5.2.4	Numerical Solution of the Nonlinear Discrete Problem ...	335
5.3	Structural Models .....	335
5.3.1	Aeroelastic Model of Vocal Folds Vibration with Two Degrees of Freedom .....	335
5.3.2	Dynamic Elasticity Problem .....	344
5.3.3	Finite Element Space Discretization of the Elasticity Problem .....	345
5.3.4	Time Discretization of the Structural Problem .....	347
5.4	Coupled FSI Problems .....	348
5.4.1	Coupled Problem of Incompressible Flow and Vocal Fold Rigid Body Model .....	348

5.4.2	Coupled Problem of Incompressible Flow Problem and Elastic Structure .....	351
5.5	Construction of the ALE Mapping .....	352
5.5.1	Analytical Definition of the ALE Mapping for Two Degrees of Freedom .....	352
5.5.2	Artificial Elasticity Problem .....	353
5.6	FSI Algorithms .....	355
5.6.1	Algorithms for Interaction of Fluid Flow and Vibrating Rigid Body .....	355
5.6.2	Algorithms for Interaction of Fluid and Elastic Structure .....	356
5.7	Numerical Examples .....	358
5.7.1	Interaction of Incompressible Flow with Rigid Body Models of Vocal Folds .....	358
5.7.2	Interaction of Incompressible Flow with Elastic Model of Vocal Folds .....	366
5.8	Interaction of Compressible Flow with Elastic Structure .....	370
5.8.1	Compressible Navier–Stokes Equations .....	371
5.8.2	ALE Form of Compressible Navier–Stokes Equations ....	372
5.9	Discretization of Viscous Compressible Flow .....	372
5.9.1	Discontinuous Galerkin Space Semidiscretization .....	373
5.9.2	Time Discretization by the BDF Method .....	375
5.9.3	Semi-implicit BDF Scheme .....	376
5.9.4	Realization of the Boundary Conditions .....	377
5.9.5	Stabilization by the Local Artificial Viscosity .....	379
5.9.6	Coupling Procedure .....	381
5.10	Numerical Results Obtained by the Discontinuous Galerkin Method .....	382
5.10.1	Flow in a Channel with Prescribed Motion of Walls .....	382
5.10.2	Interaction of Compressible Flow with a Model of Elastic Vocal Folds .....	384
5.11	Conclusion .....	387
5.11.1	Discussion of the Results .....	387
5.11.2	Open Problems and Topics for Future Work .....	388
	References .....	388

## 6 Data Assimilation in Cardiovascular Fluid–Structure

	<b>Interaction Problems: An Introduction</b> .....	395
	Luca Bertagna, Marta D’Elia, Mauro Perego, and Alessandro Veneziani	
6.1	Preliminaries .....	396
6.1.1	An Introductory Example .....	400
6.2	Probabilistic Approach .....	402
6.2.1	Basic Notation and Concepts .....	404
6.2.2	Minimum Variance and Other In-Out Estimators .....	406

6.2.3	The Kalman Filter for Linear Problems .....	412
6.2.4	Extension of the Kalman Filter to Nonlinear Problems ....	423
6.2.5	The Unscented Kalman Filter .....	426
6.3	Deterministic Variational Assimilation Methods .....	428
6.3.1	Least Squares Estimators .....	430
6.3.2	Constrained Minimization Problems with PDEs: A Simple Working Example .....	432
6.3.3	Reducing the Costs Via Solution Reduction .....	445
6.4	Some Applications of Data Assimilation in Hemodynamics Problems .....	449
6.4.1	Assimilation of Velocity Measures in Blood Flow Simulations .....	449
6.4.2	Estimation of the Arterial Compliance from Measurements of Displacement: An Inverse Fluid–Structure Interaction Problem .....	460
6.5	Conclusions .....	476
	References .....	477
<b>7</b>	<b>Mathematical Models for Blood Coagulation</b> .....	<b>483</b>
	Tomáš Bodnár, Antonio Fasano, and Adélia Sequeira	
7.1	Introduction .....	483
7.2	Historical Remarks .....	484
7.3	Cells and Proteins Intervening in the Formation and Dissolution of Clots .....	489
7.3.1	Blood Cells .....	489
7.3.2	Platelet Regulators .....	491
7.3.3	The Coagulation “Factors” .....	492
7.3.4	Fibrinolysis Factors .....	496
7.3.5	Factors Inhibitors .....	496
7.4	The Cell-Based Model for Secondary Hemostasis .....	498
7.4.1	Secondary Hemostasis .....	498
7.4.2	Fibrinolysis (Fig. 7.8) .....	503
7.5	Bleeding Disorders .....	504
7.5.1	Platelet-Related Bleeding Disorders .....	504
7.5.2	Disorders Due to Coagulation Factors Deficiency .....	506
7.5.3	Disorders Due to Proteins C, S, Z or to Vitamin K Deficiency .....	507
7.5.4	Deep Vein (or Venous) Thrombosis (DVT) .....	507
7.5.5	Heart Arrhythmia and Thromboembolism .....	508
7.5.6	Coagulation on Artificial Surfaces .....	508
7.5.7	Disseminated Intravascular Coagulation .....	509
7.6	The 3-Pathway Cascade Model .....	509

7.7 Mathematical Description of Coagulation Models ..... 511

7.7.1 Scale-Based Classification of Coagulation Models ..... 512

7.7.2 Feature-Based Classification of Coagulation Models ..... 538

7.8 Conclusions and Remarks ..... 556

References ..... 557