
Contents

<i>Preface</i>	<i>page xi</i>
<i>Course Outline</i>	<i>xv</i>
1 Introduction	1
1.1 Motivation	1
1.2 Graphs and Their Degree and Connectivity Structure	2
1.3 Complex Networks: the Infamous Internet Example	5
1.4 Scale-Free, Highly Connected and Small-World Graph Sequences	11
1.5 Further Network Statistics	16
1.6 Other Real-World Network Examples	20
1.7 Tales of Tails	40
1.8 Random Graph Models for Complex Networks	45
1.9 Notation	49
1.10 Notes and Discussion	50
1.11 Exercises for Chapter 1	52
Part I Preliminaries	55
2 Probabilistic Methods	57
2.1 Convergence of Random Variables	57
2.2 Coupling	62
2.3 Stochastic Ordering	65
2.4 Probabilistic Bounds	69
2.5 Martingales	73
2.6 Order Statistics and Extreme Value Theory	78
2.7 Notes and Discussion	82
2.8 Exercises for Chapter 2	83
3 Branching Processes	87
3.1 Survival versus Extinction	87
3.2 Family Moments	91
3.3 Random-Walk Perspective to Branching Processes	92
3.4 Supercritical Branching Processes	96
3.5 Hitting-Time Theorem and the Total Progeny	100
3.6 Properties of Poisson Branching Processes	102
3.7 Binomial and Poisson Branching Processes	107

3.8	Notes and Discussion	109
3.9	Exercises for Chapter 3	111
Part II	Basic Models	115
4	Phase Transition for the Erdős–Rényi Random Graph	117
4.1	Introduction	117
4.2	Comparisons to Branching Processes	122
4.3	The Subcritical Regime	124
4.4	The Supercritical Regime	130
4.5	CLT for the Giant Component	139
4.6	Notes and Discussion	145
4.7	Exercises for Chapter 4	146
5	Erdős–Rényi Random Graph Revisited	150
5.1	The Critical Behavior	150
5.2	Critical Erdős–Rényi Random Graphs with Martingales	156
5.3	Connectivity Threshold	164
5.4	Degree Sequence of the Erdős–Rényi Random Graph	168
5.5	Notes and Discussion	172
5.6	Exercises for Chapter 5	174
Part III	Models for Complex Networks	177
	<i>Intermezzo: Back to Real-World Networks ...</i>	179
6	Generalized Random Graphs	183
6.1	Motivation of the Model	183
6.2	Introduction of the Model	184
6.3	Degrees in the Generalized Random Graph	190
6.4	Degree Sequence of Generalized Random Graph	194
6.5	Generalized Random Graph with I.I.D. Weights	197
6.6	Generalized Random Graph Conditioned on Its Degrees	199
6.7	Asymptotic Equivalence of Inhomogeneous Random Graphs	203
6.8	Related Inhomogeneous Random Graph Models	207
6.9	Notes and Discussion	209
6.10	Exercises for Chapter 6	210
7	Configuration Model	216
7.1	Motivation for the Configuration Model	216
7.2	Introduction to the Model	218
7.3	Erased Configuration Model	227
7.4	Repeated Configuration Model: Simplicity Probability	232
7.5	Uniform Simple Graphs and Generalized Random Graphs	236
7.6	Configuration Model with I.I.D. Degrees	241
7.7	Related Results on the Configuration Model	245
7.8	Related Random Graph Models	248

7.9	Notes and Discussion	250
7.10	Exercises for Chapter 7	252
8	Preferential Attachment Models	256
8.1	Motivation for the Preferential Attachment Model	256
8.2	Introduction of the Model	259
8.3	Degrees of Fixed Vertices	262
8.4	Degree Sequences of Preferential Attachment Models	264
8.5	Concentration of the Degree Sequence	267
8.6	Expected Degree Sequence	270
8.7	Maximal Degree in Preferential Attachment Models	283
8.8	Related Results for Preferential Attachment Models	288
8.9	Related Preferential Attachment Models	290
8.10	Notes and Discussion	294
8.11	Exercises for Chapter 8	297
<i>Appendix</i>		301
<i>Glossary</i>		304
<i>References</i>		306
<i>Index</i>		317